

Quadrivium I

Lesson Plans

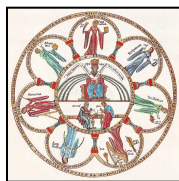
Week 8: The Golden Ratio

Overview: While last week was mostly about using geometry to understand ratio, we now use ratio to understand geometry. Specifically, we look at the famous golden ratio from two perspectives.

Cultivated Virtue: *Joyfully Obligated (Justice)*. Obligations compel us to fulfil a duty or requirement, whether we are grateful or not. However, we are **obliged** when we are bound by duty and gratitude. For example, when we say, “much obliged,” it is a traditional way of expressing gratitude. Obligations and occasions to be obliged are rooted in justice. The virtue of **justice** consists in a firm, constant, and perpetual will to render to each person what we owe them. We tend to view the obligations of justice as obstacles to our happiness. Given our fallen but redeemed nature, God gives us these obligations in the context of relationships with others because we need them. Obligations are not the obstacles but rather the true path to happiness. Gratitude transforms obstacles into opportunities. Every time we cultivate being **joyfully obliged**, we are exercising great faith and hope through gratitude and paving the way for lived charity. The Holy Spirit’s gift of **piety** perfects justice by teaching us to have a true love for God as our Father, for all human beings as his children, and for every justified obligation as the way to true joy and lasting happiness. Pray for the gift of piety this week and to cultivate being joyfully obliged to your work as a student, your role in your home, and in your calling to greatness.

Assessment Preparation: Chapter 8 will be assessed by means of one demonstrating proof and one construction. Students should be prepared to prove Proposition 52 (II.6) from memory. Students should also be able to state two ways of defining the golden ratio. Finally, students will construct a regular pentagon using compass and straightedge.

Looking Ahead: Next week the next discipline, arithmetic, begins. We will be taking advantage of the cooler weather to begin astronomical observations.



Monday

Day 1 Journal Entry: Add a suitable heading. For example, “Chapter 8 Day 1—The Golden Ratio.”

- Add an entry for Proposition 52

Proposition 52 (II.11). *The procedure given above yields a division of AB so that the square on AC is the same as the rectangle on BC and AB .*

- Complete Exercises 1.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*

Tuesday

- II. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- III. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson. We will be reading from his essay on Beauty each day this week.

Beauty

Euclid alone has looked on Beauty bare.

Edna St. Vincent Millay won fame as a poet at the age of 20, when she entered a contest sponsored by *The Lyric Year*. Her fourth-place finish in that competition was a great controversy. Ten years later, at which point she was an established name in the literary world, Millay published the poem whose first line is quoted above.

Among her elegant words about the vision to which the ancient geometer invites us, Millay also offers an admonition. She says, “let all who prate of beauty hold their peace.” A danger lurks; we can be hastily presumptuous when we speak of aesthetic delight, and by this presumption fail to rise above ourselves.

The poem continues “and lay them prone upon the earth and cease/to ponder on themselves.” This posture, prone upon the earth, features also in the contest poem, *Renascance*, that first made her name. “So here upon my back I’ll lie/ And look my fill into the sky.” In this older poem the contemplative pose generates a more ambivalent outcome. The narrator, with senses open to all that is in the world, comes to be oppressed and overwhelmed by the suffering within it. “Ah, awful weight! Infinity / Pressed down upon the finite Me!” After a death and rebirth, refreshed by rain, she experiences new joy, confident that she can find the divine presence in every thing...

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- IV. **Reading Excerpt from the Chapter:** Reread section 8.1
- Critically read and annotate section 8.2 Another Perspective.
 - Review section 7.2, focusing especially on Proposition VI.1
- V. **Day 2 Journal Entry:** Add a suitable heading. For example, “Chapter 8 Day 2—Another Perspective (1)”
- **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Add an entry for Proposition 53 (VI.14, partial).
 - Complete Exercise 2.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
- VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Wednesday (*Virtual Office Hours Day)

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson. We will be reading from his essay on Beauty each day this week.

Beauty

Euclid alone has looked on Beauty bare.

...In both poems, *Euclid alone* and *Renascence*, Millay shows that an encounter with the transcendent is not merely pleasant, though it might please. Mathematical beauty is both “luminous” and “terrible” in her account. If we would speak of beauty in the subjects we teach and learn, we do well to keep these both in mind, that the beautiful is delightful but also holy.

In the presence of something holy we rightfully fall silent. Shakespeare writes of silence in a more mercantile vein.

*My love is strengthened, though more weak in seeming,
I love not less, though less the show appear.
That love is merchandized whose rich esteeming
the owner's tongue doth publish everywhere*

The Bard notes that, however beautiful our words, they cheapen through overuse. A love that remains strong and true need not shout the fact, year after year. As the couplet has it, *I will not dull you with my song.*

This chapter, the final chapter of our study of geometry, involves a ratio whose aesthetic properties have often been touted, I neither propose nor presume any definite aesthetic responses, here or elsewhere in *A Brief Quadrivium*. There are two reasons: one is historical, the other is principled...

- III. **Reading Excerpt from the Chapter:** Review section 8.2
- Critically read and annotate section 8.3: A Remarkable Construction
 - Review the procedure from Chapter 2 on how to circumscribe a circle about a triangle.
- IV. **Day 3 Journal Entry:** Add a suitable heading. For example, “Chapter 8 Day 3—Another Perspective (2)”
- **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Complete Exercises 3 and 4.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
- V. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Thursday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson. We will be reading from his essay on Beauty each day this week.

Beauty

Euclid alone has looked on Beauty bare.

The historical reason to avoid a lengthy discussion of this ratio’s purported pulchritude is simple, such discussion is not a feature of the classical sources of the quadrivium. In Euclid, in particular, this ratio—what he calls division of a segment in “extreme and mean ratio”—is treated technically, just like the rest of his material. This special ratio allows, among other things, the construction of a regular pentagon, the construction students learn here in Chapter 8. While reasonable claims can be made about the use of the golden ratio in ancient art and architecture, it seems it was not until Luca Pacioli’s *Divina proportione*, illustrated by Leonardo da Vinci and printed in 1509, that the ratio’s relation to beauty was treated at length in any text.

The second, more principled reason has to do with student ability. Phi, the golden ratio, does indeed possess many remarkable properties, and the ratio reveals itself in created things in striking, often surprising ways. These properties and appearances, though, are often not things that typical students of *A Brief Quadrivium* are prepared to treat in a properly mathematical way. To give one example, the convergence to phi of ratios of consecutive elements in the Fibonacci sequence requires, for a genuine mathematical treatment, some theory of infinite sequences and limits. Rather than a superficial account of these fascinating matters, then, I have chosen to stick with what we can do thoroughly.

We may speak about beauty...We may find it, too, as we apply ourselves to ancient disciplines. As we do this, let us beware lest our speech lose the sobriety born of a sacred encounter.

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- III. **Reading Excerpt from the Chapter:** Review section 8.3.
 - Return to Section 8.1 and study the proof of Proposition 52 (II.11) more closely. It should be clearer with the time that has elapsed.
- IV. **Day 4 Journal Entry:** Add a suitable heading. For example, “Chapter 8 Day 4—A Remarkable Construction.”
 - **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Complete Exercises 5, 6, and 7.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
 - *Start working ahead for Astronomy. Complete exercise 11.
- V. **Assessment Prep:** Review the assessment preparation on the first page. What do you need to do before Monday to prepare to excellently demonstrate excellent work? What are you willing to do?
Make a plan. Execute plan.
 - **Practice demonstrating with a partner.** Make this an essential part of your plan.
- VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Friday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
 - II. **Opening Reading Excerpt:** Reread the cultivated virtue of the week on the first page. Reflect on how you have grown this week in that virtue.
 - III. **Reading Excerpt from the Chapter:** Reread all of Chapters 7 and 8.
 - IV. **Day 5 Journal Entry:** Add a suitable heading. For example, “Chapter 8 Day 5—The Golden Ratio.”
 - **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Complete Exercises 8, 9, and 10.
 - V. **Assessment Preparation:** Practice for the Week 8 Assessment. Prompts for the assessment are given below.
 - Work on executing your plan.
 - **Practice demonstrating with a partner.**
 - VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.
-

Week 8 Assessment

- State two ways of defining the golden ratio. One involves equal parallelograms. The other involves ratio directly.
- Given Proposition II.6 (indicated in Figure 8.2), prove Proposition 52 (II.11).
- Construct a regular pentagon using a compass and straightedge.

