

Quadrivium I

Lesson Plans

Week 6: The Composition of Quadrilaterals

Overview: The highlight of this week is the Pythagorean theorem. This significant theorem will be fundamental in later work. We will make frequent use of it when studying astronomy. The theorem is likely familiar to you; keep in mind the difference between knowing the statement and knowing the proof.

Cultivated Virtue: *Solitude (Prudence/ Temperance)*. St. Albert the Great (the teacher of St. Thomas Aquinas), lists *solitude* as one of the forty-two virtues to reach heaven in his work, The Paradise of the Soul. He writes:

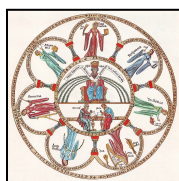
True solitude is the practice of withdrawing the mind from all exterior occupations and cares, and from all delights in created things. The person who practices this type of spiritual solitude will then direct all his affections, thoughts, and desires toward God alone, to the extent that he thereby becomes one in spirit with God.

Turns out, the kind of solitude that makes for great prayer also makes for great study. When studying difficult topics, like those of the quadrivium, the quality of our learning is directly proportional to the quality of our focus and attention. Solitude is prudent. The problem when we struggle to learn hard things is often not due to our lack of intelligence, but rather the imprudent ways we try to learn: in the presence of screens, blasting distracting music, studying under the delusion of multitasking...etc.]

This brings us to the role of temperance in seeking solitude. Being physically (or digitally) alone and in silence is not enough to have solitude. If our soul remains occupied with worldly concerns and earthly desires, we need the virtuous detachment that comes from temperance. Throughout the week, you will be assigned demanding journal entries: work that demands solitude. Cultivate solitude this week by setting aside short, but intense, times in the day for the daily journal. Ask the Lord in prayer to be more attentive to those things that destroy your solitude, and pray for the grace to be more active in eliminating those things through prudence and temperance.

Assessment Preparation: Chapter 6 will be assessed by means of demonstrating proofs. Students should be prepared to prove Proposition 34 (I.47) from memory. Students will have the option to select one of the following to prove: Proposition 28 (I.34) or Proposition 29 (I.35).

Looking Ahead: Next week takes a turn away from pure geometry, to the more general notion of ratio. Our plan follows Euclid to a degree; the Pythagorean theorem concludes Book I of Euclid's *Elements*, and ratio is Euclid's theme in Books V and VI. These are elementary but demanding topics, and the perspective of Chapter 7 is likely to be new to you.



Monday

Day 1 Journal Entry: Add a suitable heading. For example, “Chapter 6 Day 1—Parallelograms.”

- Add entries for Definition 26, Proposition 27 (I.33), Proposition 28 (I.34) and Proposition 29 (I.35) [see examples below for all such entries]

Definition 26. *A parallelogram is a figure of four sides in which a pair of opposite sides is parallel and equal.*

Proposition 27 (I.33). *In a parallelogram, each pair of opposite sides is equal and parallel.*

- Complete Exercises 1, 2, 3, and 4.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*

Tuesday

- II. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- III. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

To study *A Brief Quadrivium* in its entirety is a challenging task, one that will take some time. Things that take a long time require us to be patient. It is good to know in advance some of the points requiring patience. [from *Magister*: We will look at just the third one today and the fourth one tomorrow. Yes. Again. These are the same as last week...and the week before. They are verily, indeed that important!]

- Practicing the actions.
- Going from actions to explanations.
- **Understanding the statement given.**
- Understanding the proof of a statement.
- Understanding how the various statements form a single, structured body of thought.
- Understanding how the various disciplines—geometry, arithmetic, music, astronomy—themselves amount to one single thing, one way composed of four ways.

Understanding the statement given: The early propositions that we prove will be fairly straightforward and clear. Later assertions are more obscure. How do we shed light? Consider these questions when reading a statement. Do we understand all of the terms? Do we distinguish between the hypothesis and the conclusion? Do we have some grasp of a relation between the things given and the things sought? After patient work, reviewing, rereading, reconsidering, we eventually grasp the statement considered alone. The task of understanding is not complete, as clarified by the next couple of items in tomorrow’s reading.

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- IV. **Reading Excerpt from the Chapter:** Revisit your outline and inspectional reading of Chapter 6.
- Critically read and annotate the remainder of section 6.2: Equality of Parallelograms.
 - Notice that the notion of “equality” indicated in Proposition 29. How is this different from what we did before? Sameness for triangles was used in a strict sense in Chapter 4. The sameness or equality we use here is looser. This will be the case for Proposition 29, but more so will it be the case for Proposition 30. (See the opening reading for Thursday for more. Also, a further discussion on this will accompany the video for Proposition 30.)
- V. **Day 2 Journal Entry:** Add a suitable heading. For example, “Chapter 6 Day 2—Equality of Parallelograms.”
- **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Add an entry for Proposition 30 (I.37) and Proposition 31 (I.41) .
 - Complete Exercises 5 and 6.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
- VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Wednesday (*Virtual Office Hours Day)

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

To study *A Brief Quadrivium* in its entirety is a challenging task, one that will take some time. Things that take a long time require us to be patient. It is good to know in advance some of the points requiring patience. [from *Magister*: We will look at just the fourth one today. Again, same as last week and the one before. Seriously, this point is indeed that important.]

- Practicing the actions.
- Going from actions to explanations.
- Understanding the statement given.
- **Understanding the proof of a statement.**
- Understanding how the various statements form a single, structured body of thought.
- Understanding how the various disciplines—geometry, arithmetic, music, astronomy—themselves amount to one single thing, one way composed of four ways.

Understanding the proof of a statement: Understanding a mathematical statement is just the beginning. The heart of mathematics is in the proof, the explanation of why a statement follows from others that have been accepted. It takes time to understand a proof. How does this one thing follow from a previous one? Am I forgetting something important? How can I keep all the details in mind? Is this account really persuasive, compelling, complete? These questions will come to mind as you wrestle with mathematical proofs.

- III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 6.
- Critically read and annotate section 6.3: The Pythagorean Theorem.
 - Exercise **heroic patience** to critically and carefully study Theorem 34 (I.47).
- IV. **Day 3 Journal Entry:** Add a suitable heading. For example, “Chapter 6 Day 3—The Pythagorean Theorem (1)”
- **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Add an entry for Definition 32, Definition 33, and Theorem 34 (I.47)
 - Complete Exercises 8.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
 - Attempt Exercise 7 (on scratch paper)
- V. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Thursday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

Analogy

If a word is used in entirely different ways, we say that it is used “equivocally,” and if used in the exact same way we say that it is used “univocally.” Between univocal and equivocal uses, there is an intermediate place. In that case, we say the words are used “analogically.” This means that the use of the words are related, but still different.

In the study of four-sided shapes, we encounter a notable instance of analogical speech in the words “same” and “equal.” Sameness for triangles, in Chapter 4, was a fairly strict, rigid notion. The sameness of or equality we use for quadrilaterals is looser. This looser meaning also makes sense for triangles, and we use it in Proposition I.37, and not the stricter one.

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 6.
- Reread Section 6.3 again, paying careful attention to all details in the proof of the Pythagorean theorem.
- IV. **Day 4 Journal Entry:** Add a suitable heading. For example, “Chapter 6 Day 4—The Pythagorean Theorem (2).”
- **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Complete Exercise 8 Again.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
 - Attempt Exercise 9
- V. **Assessment Prep:** Review the assessment preparation on the first page. What do you need to do before Monday to prepare to excellently demonstrate excellent work? What are you willing to do? **Make a plan. Execute plan.**
- **Practice demonstrating with a partner.** Make this an essential part of your plan.
- VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Friday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
 - II. **Opening Reading Excerpt:** Reread the cultivated virtue of the week on the first page. Reflect on how you can grow in solitude.
 - III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 6..
 - Review carefully the proofs of Proposition 29 (I.35) and Theorem 34 (I.47)
 - Practice demonstrating Theorem 34 (I.47).
 - IV. **Day 5 Journal Entry:** Add a suitable heading. For example, “Chapter 6 Day 5—The Pythagorean Theorem (3).”
 - **Cultivated Virtue: Solitude.** Seek solitude while working on your journal entry.
 - Construct the diagram for Theorem 34 (I.47) using your instruments (see page 46 for reference)
 - V. **Optional Enrichment:** Try out any remaining exercises that are of interest to you from Chapter 6.
 - VI. **Assessment Preparation:** Practice for the Week 4 Assessment. Prompts for the assessment are given below.
 - Work on executing your plan.
 - **Practice demonstrating with a partner.**
 - VII. **Close in Prayer:** As with all things, conclude with gratitude in prayer.
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Week 6 Assessment

- Demonstrate a proof for Proposition 34 (I.47). Justify statements in the proof with reference to definitions, postulates, and/or other propositions.
- Prove Proposition 28 (I.34) or Proposition 29 (I.35) (select one).

