

Quadrivium I

Lesson Plans

Week 4: Proofs with Triangles

Overview: The fourth chapter offers the first proofs of the book, involving the simplest geometric shapes.

Cultivated Virtue: *Studiosness (Prudence)*. The reflection this week comes from our author, Dr. Peter Ulrickson.

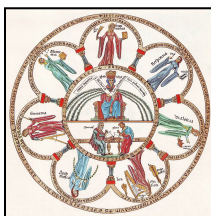
Prudence is the virtue by which we choose fitting means to attain a good end. The means that lead to a good end depend on many circumstances that vary with place and time, and so prudence cannot be reduced to theorizing about wholly abstract cases, divorced from all particulars. Prudence is inescapably personal. While there can be discussion about the prudence of a course of action—and indeed taking suitable counsel is often a part of prudence—this deliberation never eliminates a fundamental hiddenness and obscurity in the judgment of prudence.

[from *Magister*: St. Thomas Aquinas discusses *studiositas* as a virtue under temperance, along with *curiositas*, its corresponding vice. What we mean by studiosness is a little different. Under temperance, *studiositas* refers to the virtuous pursuit of learning as both moderate and rightly motivated. Under prudence, we refer to the virtuous methods of learning as the fitting means to attain the good of learning. It is not enough for us to do good things. We must do good things, in the right way, and for the right reason. Cramming and other myopic (short-sighted) learning techniques are not fitting for the dignity of the human person, as they produce short-term gain at the cost of long-term formation.

Prudence in the life of a student, *studiousness*, is hard for the same reason prudence is hard—it is inescapably personal. Rely on the methods given to you in these daily plans. They are forged with your studiosness in mind, your long-term formation: the cultivation of wisdom and virtue that is the goal of education in the liberal arts.]

Assessment Preparation: Chapter 4 will be assessed by means of demonstrating proofs. Students should be prepared to prove Proposition 9 (I.1). Students will have the option to select one of the following to prove: Proposition 14 (I.5) or Proposition 15 (I.6).

Looking Ahead: Taking stock of where we have been: we developed skill in the use of our instruments, we applied that skill to basic geometric constructions, and we moved from doing to thinking by means of getting in the middle of things with terms, postulates, and proofs. We only use the first three postulates right now. The fourth and fifth postulates will be used in the next couple of chapters.



Monday

Day 1 Journal Entry: Add a suitable heading. For example, “Chapter 4 Day 1—A First Proof.”

- Add entries for Proposition 9 (I.1) and Proposition 10 (I.1). See examples below.

Proposition 9 (I.1). *On a given segment to construct an equilateral triangle.* [Proof with table]

Proposition 10 (I.1). *On a given segment to construct an equilateral triangle.* [More succinct proof]

- Complete Exercises 1, 2, and 4.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*

Tuesday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

To study *A Brief Quadrivium* in its entirety is a challenging task, one that will take some time. Things that take a long time require us to be patient. It is good to know in advance some of the points requiring patience. [from *Magister*: We will look at just the third one today and the fourth one tomorrow.]

- Practicing the actions.
- Going from actions to explanations.
- **Understanding the statement given.**
- Understanding the proof of a statement.
- Understanding how the various statements form a single, structured body of thought.
- Understanding how the various disciplines—geometry, arithmetic, music, astronomy—themselves amount to one single thing, one way composed of four ways.

Understanding the statement given: The early propositions that we prove will be fairly straightforward and clear. Later assertions are more obscure. How do we shed light? Consider these questions when reading a statement. Do we understand all of the terms? Do we distinguish between the hypothesis and the conclusion? Do we have some grasp of a relation between the things given and the things sought? After patient work, reviewing, rereading, reconsidering, we eventually grasp the statement considered alone. The task of understanding is not complete, as clarified by the next couple of items in tomorrow’s reading.

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 4, paying special attention to questions that may still arise and those that seem clearer when seeing it for a second time. (Be sure to send me an email about persisting questions, or plan to bring it up in our Wednesday check-in).
- Critically read and outline Section 4.3.
- IV. **Day 2 Journal Entry:** Add a suitable heading. For example, “Chapter 4 Day 2—Accepted Propositions.”
- Add entries for Proposition 11 (I.3) and Proposition 12 (I.4).
 - Attempt Exercise 3 (Be prepared to discuss this with me on Wednesday). Complete Exercise 6.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
- V. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Wednesday (*Virtual Office Hours Day)

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

To study *A Brief Quadrivium* in its entirety is a challenging task, one that will take some time. Things that take a long time require us to be patient. It is good to know in advance some of the points requiring patience. [from *Magister*: We will look at just the fourth one today.]

- Practicing the actions.
- Going from actions to explanations.
- Understanding the statement given.
- **Understanding the proof of a statement.**
- Understanding how the various statements form a single, structured body of thought.
- Understanding how the various disciplines—geometry, arithmetic, music, astronomy—themselves amount to one single thing, one way composed of four ways.

Understanding the proof of a statement: Understanding a mathematical statement is just the beginning. The heart of mathematics is in the proof, the explanation of why a statement follows from others that have been accepted. It takes time to understand a proof. How does this one thing follow from a previous one? Am I forgetting something important? How can I keep all the details in mind? Is this account really persuasive, compelling, complete? These questions will come to mind as you wrestle with mathematical proofs.

- III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 4.
- Review the statements of Propositions 9 (I.1), 11 (I.3), and 12 (I.4). Be sure you have arrived at an understanding of the statement given as described in yesterday’s opening reading.
 - Critically read and outline the fourth section, *Isosceles Triangles*, of the fourth chapter, at least through the conclusion of the proof of Proposition 14 (I.5).
 - Carefully work through Proposition 14 (I.5). Observe that it is not possible to keep everything in the mind at once in such a long proof. Instead, we must proceed step by step, marking our progress as we go. It can be helpful to add marks to a diagram to indicate things that have been shown to be equal. ***Exercise heroic patience!*** Follow the guidance in the opening reading.
- IV. **Day 3 Journal Entry:** Add a suitable heading. For example, “Chapter 4 Day 3—Isosceles Triangles.”
- Add Definition 13 and Proposition 14 (I.5).
 - Copy the proof of Proposition 14 (I.5). Include a labeled diagram.
 - Attempt Exercise 7.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
- V. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Thursday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
- II. **Opening Reading Excerpt:** Carefully and critically read the following excerpt from Dr. Peter Ulrickson.

Speech

A proof can be treated as a purely logical entity. Does the asserted relation between the hypothesis and the conclusion indeed hold? Does the account of this relation given in the proof succeed in exposing the nature of the relation? These are, in a sense, absolute questions, questions whose answer requires no context.

A proof is also relational, though; it is a rhetorical act and not simply a logical one. Euclid’s Proposition I.1 has a number of proofs. Are any of them wrong? No. Are any of them the best? No. The fittingness of the proof cannot be judged in the abstract. We must consider it as something designed to communicate to some person.

[from *Magister*: This is why the quadrivium follows the trivium. Bring everything you have gained from the trivium about beauty in words to bear on your demonstration of proofs. The grammar, logic, and rhetoric of the trivium are meant to free the mind through language to behold great things and to effectively share that encounter with others. Truth is great (even if it is about a triangle). Try to enjoy the human element of the quadrivium: connecting with others over encountering truth.]

Ponder this excerpt as you complete your work. Be sure to jot down any interesting questions or thoughts you’d like to share during the next class meeting.

- III. **Reading Excerpt from the Chapter:** Review the proof of Proposition 14 (I.5) with heroic fortitude (patience).
- Critically read the remainder of section 4, Isosceles Triangles. Starting with the paragraph on p. 26, “The definition of isosceles...,” continue reading until the end of the section.
 - Pay close attention to the idea of *reductio ad absurdum*. Consider this example: “Suppose mom were at the store. Then she would have taken the van. But the van is here. Therefore...” Non-mathematical examples will, of course, not be perfectly demonstrative.
 - Consider a statement and its converse. One example: if a shape is a square, then it is a rectangle. Another example: if a person is a resident of New Jersey, then he is a resident of the United States. Observe that I.5 and I.6 are converses. Each is a distinct assertion, each of which requires its own proof.
 - Read Section 4.5, Some Additional Results, through the proof of Proposition 18 (I.13).
- IV. **Day 4 Journal Entry:** Add a suitable heading. For example, “Chapter 4 Day 4—Isosceles Triangles Continued.”
- Add entries for Proposition 15 (I.6), Proposition 16 (I.7), Proposition 17 (I.8), and Proposition 18 (I.13).
 - Complete Exercises 8, 10, and 12.
 - *Be sure to write the prompt for each exercise.*
 - *Use impeccable grammar: Be clear, concise, accurate, and eloquent.*
 - (optional) Complete Exercises 9 and 11. (Optional means you should complete these if prudent: time permitting and need permitting.)
- V. **Assessment Prep:** Review the assessment preparation on the first page. What do you need to do before Monday to prepare to excellently demonstrate excellent work? What are you willing to do? **Make a plan. Execute plan.**
- VI. **Close in Prayer:** As with all things, conclude with gratitude in prayer.

Friday

- I. **Recitation/ Prayer:** Begin by reciting the “Rules of a Virtuous Classroom” and praying “A Prayer Before Study” of St. Thomas Aquinas
 - II. **Opening Reading Excerpt:** Reread the cultivated virtue of the week on the first page. Reflect on how you can grow in studiousness.
 - III. **Reading Excerpt from the Chapter:** Revisit your inspectional reading of Chapter 4.
 - Critically read the remainder of section 5, *Some Additional Results*.
 - IV. **Day 5 Journal Entry:** Add a suitable heading. For example, “Chapter 4 Day 5—Some Additional Results.”
 - Add entries for Proposition 19 (I.15) and Proposition 20 (I.16).
 - Complete Exercise 17.
 - V. **Optional Enrichment:** Try out any remaining exercises that are of interest to you.
 - VI. **Assessment Preparation:** Practice for the Week 4 Assessment. Prompts for the assessment are given below.
 - VII. **Close in Prayer:** As with all things, conclude with gratitude in prayer.
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Week 4 Assessment

- Prove Proposition 9 (I.1). Justify statements in the proof with reference to definitions and postulates.
- Prove Proposition 14 (I.5) or Proposition 15 (I.6) (select one).

